Disequilibrium Incentives in Resource Allocation Conflicts

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Abstract

This paper experimentally investigates conflicts where resources are allocated over multiple contests to compete for shares of complementary factors. A competitor's share of a given factor is proportional to a power function of their allocation to the corresponding contest. Objective functions exhibit constant subunitary elasticity between factors. More responsive contest success functions strengthen incentives to closely approximate best responses, but also bring non-equilibrium best responses further from equilibrium predictions. Observed resource allocations were significantly closer to equilibrium predictions under less responsive success functions, consistent with best responses to non-equilibrium behavior. These results suggest that non-equilibrium incentives provide useful information about the reliability of equilibrium predictions.

Keywords: Conflict, Resource Allocation, Complementary Factors

JEL Classification: C72, C92, D74

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1 Introduction

Strategic interaction frequently involves the allocation of limited resources across multiple domains of competition. Military conflicts often involve competition for control over both airspace and the ground below it. Similarly, ride sharing platforms compete for both riders and drivers. Different domains of competition often provide complementary inputs to a decision maker's overall objective. The value of control over airspace may depend on a military faction's level of control over the ground below it (Pirnie et al., 2005). Similarly, the value of an additional rider to a ride sharing platform may depend on the platform's success in attracting drivers (Rochet and Tirole, 2003).

This paper reports an experimental study of conflicts where resources are allocated over multiple contests to compete for shares of complementary factors. A competitor's share of a given factor is proportional to a power function of their resource allocation to the corresponding contest. Factors serve as complementary inputs to a competitor's objective function.

Competitors in such conflicts face stronger incentives to closely approximate best responses when contest success functions are more responsive to investment levels. Consequently, one may expect resource allocations to approximate equilibrium predictions more closely under more responsive contest success functions. Conversely, best responses to non-equilibrium resource allocations are farther from equilibrium predictions under more responsive contest success functions. Accordingly, one may expect resource allocations to approximate equilibrium predictions more closely under less responsive contest success functions.

To test these hypotheses, the experimental design varies the responsiveness of contest success functions across treatment conditions. Equilibrium predictions are identical under every treatment condition, but observed resource allocations were significantly closer to equilibrium under less responsive success functions. These results are consistent with best responses to non-equilibrium behavior, suggesting that non-equilibrium incentives contain important information about the reliability of equilibrium predictions.

This paper contributes to the experimental literature on Blotto contests where competitors allocate limited resources to compete over multiple prizes. Much of the previous literature focuses on Blotto contests with indivisible prizes where resource allocations influence the probability of winning a given prize. Duffy and Matros (2017) find support for equilibrium predictions regarding allocation behavior in stochastic Blotto contests with winner-take-all battles and majoritarian objective functions. Chowdhury et al. (2021) find that subjects over-allocate resources to battlefields with distinctive labels in stochastic Blotto contests with winner-take-all battles. In contrast, the present paper investigates of Blotto contests for shares of complementary factors.

There is also a significant body of experimental research on contests where agents compete for a single prize. Baik et al. (2020) experimentally identify a non-monotonic relationship between budget constraints and average bids. Llorente-Saguer et al. (2023) report experimental support for theoretical predictions that bid-caps and tie-breaking rules can increase total expenditure in contests with heterogeneous contestants. A survey of this literature is provided by Cason et al. (2020).

The present paper contributes to the experimental literature investigating strategic features that influence the reliability of equilibrium predictions. Cason et al. (2014) observe behavior that is closer to equilibrium in evolutionarily stable games. The conflicts investigated by the present study are evolutionarily stable in all treatments, so evolutionary stability does not explain the observed treatment effects. Chen and Gazzale (2004) observe behavior that is closer to equilibrium in supermodular compensation mechanisms. The conflicts investigated by the present study are supermodular in all treatments, so supermodularity does not explain the observed treatment effects. Stephenson (2022) observes behavior that is closer to equilibrium in school choice mechanisms with high frequency feedback. The present study provides feedback at the same frequency in all treatments, so feedback frequency does not explain the treatment effects observed in the present study.

The remainder of this paper is organized as follows. Section 2 presents the theory. Section 3 discusses the experimental design. Section 4 describes the hypotheses. Section 5 presents the results and section 6 concludes.

2 Theory

Consider a conflict where two competitors simultaneously allocate a fixed budget between two contests. Let $x_{ik} \in \mathbb{R}_+$ denote the share of competitor i's resources allocated to contest k. As in the Blotto contest of Borel (1921), total resource investments are sunk before competitors allocate them between contests. Let X_i denote the set of all allocations $x_i \in \mathbb{R}_+^2$ such that $x_{i1}+x_{i2} = 1$. The success function $y_{ik}(x)$ describes competitor i's share of factor k as a function of the allocation profile $x \in \mathbb{R}_+^{2\times 2}$. If $x_{1k} = x_{2k} = 0$, then factor k is divided evenly between the two competitors. If $x_{1k}+x_{2k} > 0$ then the success function takes the generalized Tullock (1980) form under which competitor i's share of factor k is proportional to a power function of their allocation to contest k.

$$y_{ik}(x) = \frac{x_{ik}^{\alpha}}{x_{ik}^{\alpha} + x_{jk}^{\alpha}} \tag{1}$$

The parameter α describes the responsiveness of the success function $y_{ik}(x)$ to the allocation levels x_{ik} and x_{jk}. If α is very large then nearly the entirety of factor k is awarded to the competitor who allocates the most resources to contest k. If α is very small then factor shares are largely insensitive to resource allocations. Let $v_k \in [0,1]$ denote the relative value of factor k such that $v_1 + v_2 = 1$. Let $\pi_i(x)$ denote competitor i's objective function. Factor shares are complementary inputs to competitor i's objective function $\pi_i : X \to \mathbb{R}$. If $y_{ik}(x) = 0$ then $\pi_i(x) = 0$. If $y_i(x) \in \mathbb{R}_{++}^2$ then competitor i's payoff is given by

$$\pi_i(x) = \beta \left(v_1 y_{i1}(x)^{-c} + v_2 y_{i2}(x)^{-c}\right)^{-\frac{1}{c}} \tag{2}$$

The parameter $c > 0$ indexes the level of complementarity between factors. In the limit as c becomes large, factors are perfect complements. The elasticity of substitution between factors is given by $\eta = (1+c)^{-1} < 1$.

Strategic interaction frequently involves competition over divisible complementary factors. For example, military conflicts may involve competition for control over both airspace and the ground below it. The value of additional control over the airspace in a given region may depend on a military faction's level of control over the ground below it (Pirnie et al., 2005). Similarly, ride sharing platforms simultaneously compete for both riders and drivers. The value of an additional rider may depend on a firm's success in attracting divers (Rochet and Tirole, 2003).

This is not a zero sum game because the total payoff $\pi_1(x)+\pi_2(x)$ varies with the strategy profile x. Consider the simple case where $v_1 = v_2 = \frac{1}{2}$ and $\alpha = c = 1$. If both competitors select identical resource allocations then $x_1 = x_2$ and $\pi_1(x) = \pi_1(x) = \frac{1}{2}\beta$, so the total payoff is given by $\pi_1(x) + \pi_2(x) = \beta$. In contrast if $x_1 = \left(\frac{1}{4}, \frac{3}{4}\right)$ and $x_2 = \left(\frac{3}{4}, \frac{1}{4}\right)$ then $\pi_1(x) = \pi_2(x) = \frac{3}{8}\beta$, so the total payoff is given by $\pi_1(x) + \pi_2(x) = \frac{3}{4}\beta$.

Theorem 1 says that the equilibrium allocation to contest k is proportional to the value of prize k under unit complementarity. A proof of this theorem provided in the appendix. A characterization of equilibrium for the general case with n players, m contests, arbitrary endowments, and arbitrary complementarity is provided by Stephenson (2024).

Theorem 1. *If $c = 1$ then $x_{ik} = v_k$ in equilibrium.*

Figure 1 illustrates the equilibrium payoff function for the resource allocation game with $c = 1$, $\beta = 28$, and $v_1 = x_{j1} = 0.8$. The horizontal axis indicates competitor i's investment in contest 1 and the vertical axis indicates competitor i's payoff. The dashed line illustrates competitor i's payoff function when $\alpha = 1$. The solid line illustrates competitor i's payoff function when $\alpha = 8$. The dotted line indicates competitor j's allocation to contest 1. In both cases, the equilibrium allocation is $x_{i1} = 0.8$ and the equilibrium payoff is $\pi_i(x) = 0.5$.

Figure 2 illustrates the best response correspondence. The horizontal axis indicates competitor j's allocation to contest 1 and the vertical axis indicates competitor i's optimal allocation to contest 1. The dashed line illustrates competitor i's best response correspondence when $\alpha = 1$. The solid line indicates competitor i's best response correspondence when $\alpha = 0.8$. The dotted line indicates the equilibrium allocation to contest 1. If competitor j selects a non-equilibrium allocation, then competitor i's best response is

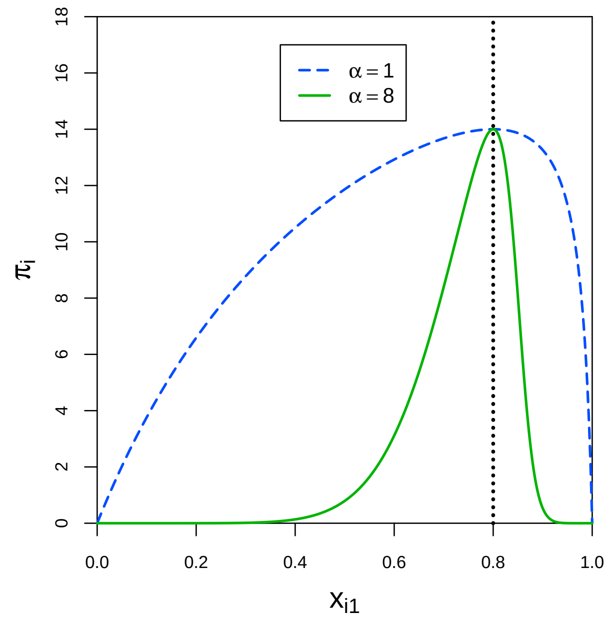

Figure 1: Equilibrium payoff functions for $c = 1$, $\beta = 28$, and $v_1 = x_{j1} = 0.8$

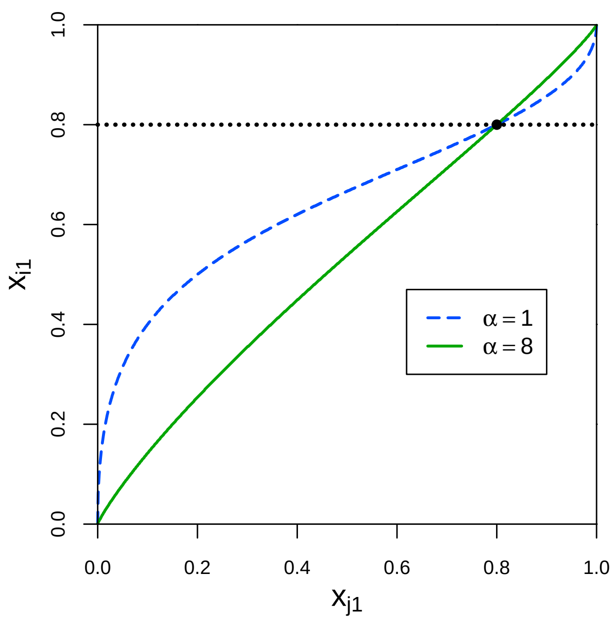

Figure 2: Best response functions for $c = 1$, $\beta = 28$, and $v_1 = 0.8$

always closer to equilibrium under $\alpha = 1$ than $\alpha = 8$.

A Nash equilibrium is said to be evolutionarily stable if small deviations from equilibrium always give the equilibrium strategy a higher payoff than than the deviating strategy (Taylor and Jonker, 1978). More formally, a symmetric Nash equilibrium (σ^*, σ^*) is said to be evolutionarily stable if, for any nonequilibrium mixed strategy $\sigma \neq \sigma^*$ and any sufficiently small $\varepsilon > 0$, $\pi_1(\sigma, \bar{\sigma}) < \pi_1(\sigma^*, \bar{\sigma})$ where $\bar{\sigma} = \varepsilon\sigma + (1-\varepsilon)\sigma^*$ denotes a mixed strategy that involves utilizing the the nonequilibrium strategy σ with probability ε and utilizing the equilibrium strategy σ^* with probability $1-\varepsilon$. Intuitively, such equilibria are stable because rare deviations from equilibrium never incentivize equilibrium players to adopt the deviating strategy. As shown in the proof of Theorem 1, the objective function π_i is strictly quasiconcave in x_i, so the Nash equilibrium x^* is always strict and the equilibrium strategy is always evolutionarily stable.

A symmetric two player game with a one dimensional strategy space is said to be supermodular if the marginal payoff to one player from increasing their strategy is increasing in the other player's strategy. The resource allocation game described above is symmetric and the strategy space is given by the one dimensional unit simplex. If $c = 1$ then differentiating player i's marginal benefit from allocation to contest 1 with respect to competitor j's allocation to contest 1 yields

$$\frac{\partial^2 \pi_i}{\partial x_{i1} \partial x_{j1}} = \alpha\beta\pi_i(x)^2 \left[\frac{v_1}{x_{i1}^2} + \frac{v_2}{x_{i2}^2}\right] \tag{3}$$

This expression is strictly positive for all $x_i \in \mathbb{R}^2_{++}$, so player i's marginal benefit from allocating resources to contest 1 is increasing in competitor j's allocation to contest 1. Hence the resource allocation game with $c = 1$ is supermodular for all $\alpha > 0$.

3 Experimental Design

Experimental sessions implemented the resource allocation game described in section 2. Each session implemented one of the four treatment conditions provided in table 1. A total of 8 experimental sessions were conducted, 2 for each of the 4 treatment conditions. Each of the 8 sessions had 20 subjects for a total of 160 experimental subjects. At the beginning of each session, subjects were randomly matched into pairs which remained fixed for the entire session. Each experimental session consisted of 100 periods. During each period, subjects allocated 100 tokens between two contests.

Valuation treatments were constructed symmetrically to control for the possibility of labeling or ordering biases. One factor value $v_i = 0.8$ and the other had value $v_j = 0.2$. In the first valuation treatment, the first factor was more valuable, so the valuation vector was given by $v = (0.8, 0.2)$. In the second valuation treatment, the second factor was more valuable, so the valuation vector was given by $v = (0.2, 0.8)$. Under both valuation treatments, the unique Nash equilibrium has competitors allocating 80% of their resources to the contest over the high value factor and 20% of their resources to the contest over the low value factor.

In the low responsiveness treatment, the responsiveness of the success function was set at $\alpha = 1$. In the high responsiveness treatment, the responsiveness of the success function was set at $\alpha = 8$. In all treatments, payoffs were determined by the objective function (2) with $c = 1$ and $\beta = 28$. At the end of each experimental session, subjects received their average payoff over all periods plus a \$7 participation bonus. Average earnings were \$19.92 per subject.

Figure 3 depicts the experimental interface. The horizontal axis indicates the number of tokens invested in contest 1 and the vertical axis indicates the subject's payoff. The blue line indicates the number of tokens the subject chooses to invest in contest 1. The black line indicates the number of tokens they invested in contest 1 during the previous period. Numerical information about allocations and payoffs are shown below the graph.

	Low Responsiveness	High Responsiveness
First Valuation	$\alpha = 1,\ v_1 = 0.8$	$\alpha = 8,\ v_1 = 0.8$
Second Valuation	$\alpha = 1,\ v_1 = 0.2$	$\alpha = 8,\ v_1 = 0.2$

Table 1: Experimental Design

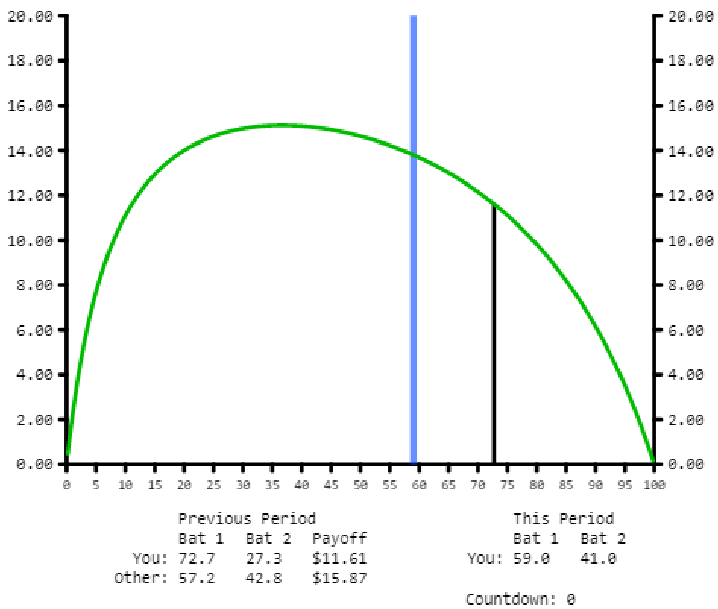

Figure 3: Experimental Interface

4 Hypotheses

As shown in Figure 2, best responses to non-equilibrium resource allocations are closer to equilibrium predictions when contest success functions are less responsive to investment levels. Accordingly, we might expect to observe outcomes that are more consistent with equilibrium predictions when contest success functions are less responsive to resource investment levels.

Hypothesis 1. *Resource allocations will be closer to equilibrium predictions in conflicts with less responsive success functions.*

As shown in Figure 1, competitors have a stronger incentive select allocations that closely approximate best responses when contest success functions are more responsive to investment levels. Accordingly, we might expect resource allocations to be more consistent with equilibrium predictions when the contest success function is more responsive to resource investment levels.

Hypothesis 2. *Resource allocations will be closer to equilibrium predictions in conflicts with more responsive success functions.*

In equilibrium, resources are allocated to each contest in proportion the value of it's factor. All experimental treatments share a unique Nash equilibrium where 80% of resources are invested in the contest over a high value factor and 20% resources are invested in the contest over a low value factor.

Hypothesis 3. *More resources will be allocated to the high value contest than the low value contest.*

5 Results

Figure 4 illustrates the empirical cumulative distribution function for distance from equilibrium under each responsiveness treatment. Distance from equilibrium is defined as the absolute difference between an observed resource allocation and the equilibrium resource allocation. The horizontal axis indicates distance from equilibrium. The vertical axis indicates the percent of distances from equilibrium at or below a given level. The solid line is the em-

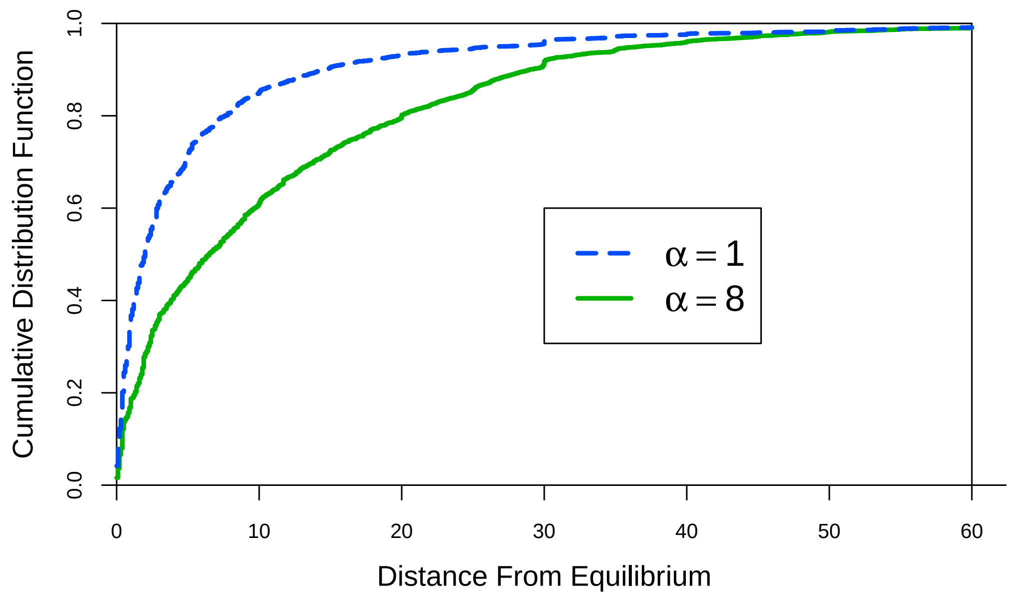

Figure 4: CDF of Distance from Equilibrium

pirical cumulative distribution function for the low responsiveness treatment where $\alpha = 1$. The dashed line is the empirical cumulative distribution function for the high responsiveness treatment where $\alpha = 8$. As shown in Figure 4, the distribution of distances from equilibrium in the high responsiveness treatment first-order stochastically dominates the distribution of distances from equilibrium in the low responsiveness treatment.

Result 1. *Resource allocations were significantly closer to equilibrium predictions in the low responsiveness treatment than the high responsiveness treatment.*

Observed resource allocations were significantly closer to equilibrium predictions under less responsive contest success functions. This result is consistent with Hypothesis 1, but inconsistent with Hypothesis 2. The average deviation from equilibrium in the low responsiveness treatment was 5.73 while the average deviation from equilibrium in the high responsiveness treatment was 11.21. As reported in table 2, both a t-test and a non-parametric Wilcoxon rank-sum test find this difference to be statistically significant at the 1% level. In both of these tests, the average allocation selected by a fixed matching

			p-value	
	$\alpha = 1$	$\alpha = 8$	rank-sum	t-test
Distance from Equilibrium	5.73	11.21	< 0.0001	< 0.0001
	Low Value	High Value	signed-rank	t-test
Average Allocation	26.6	73.4	< 0.0001	< 0.0001

Table 2: Hypothesis Tests

pair over an entire experimental session is treated as a single observation, yielding a total of 40 observations with 20 observations per treatment.

More responsive contest success functions strengthen incentives to closely approximate a best response, as illustrated in Figure 1. However, best responses need not coincide with equilibrium predictions. As illustrated in Figure 2, best responses to non-equilibrium allocations are farther from equilibrium predictions under more responsive success functions in this conflict. These non-equilibrium incentives may explain why subjects deviated farther from equilibrium in conflicts with more responsive success functions.

Figure 5 illustrates the empirical cumulative distribution function for resource allocations to the contest over high value factors. The horizontal axis indicates the percent of a subject's resources invested in the contest over the high value factor. The vertical axis indicates the percent of observed allocations to the high value contest at or below the given level. The solid line indicates the empirical cumulative distribution function for the low responsiveness treatment where $\alpha = 1$. The dashed line is the empirical cumulative distribution function for the high responsiveness treatment where $\alpha = 8$. The dotted line indicates the predicted share of resources allocated to the high value contest in equilibrium.

Result 2. *More resources were allocated to the high value contest than the low value contest.*

Consistent with Hypothesis 3, subjects allocated significantly more resources

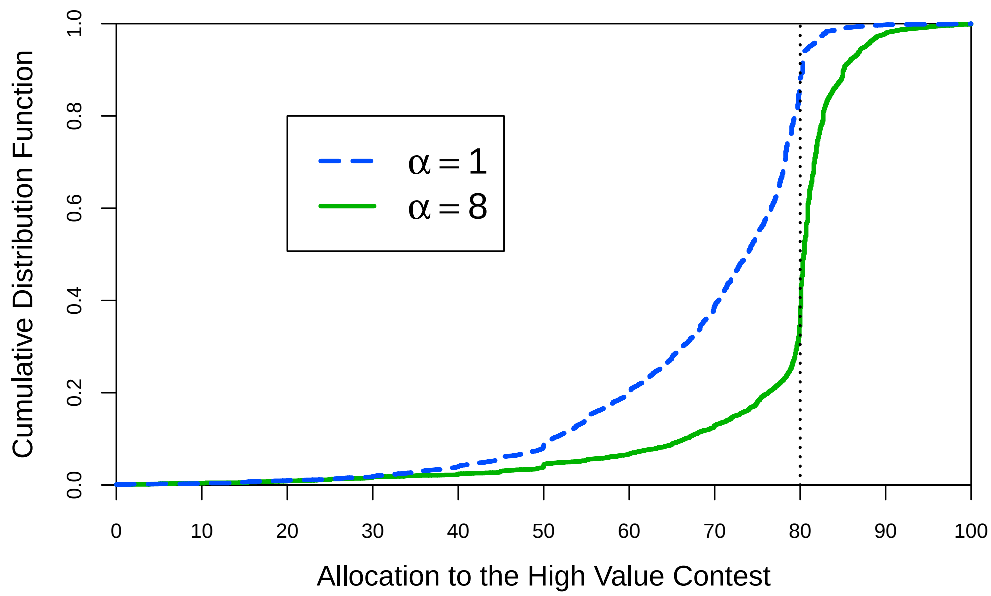

Figure 5: CDF of Investment in the High Value Contest

to contests over high value factors than contests over low value factors. On average, subjects allocated 73.4% of their resources to contests over high value factors and 26.6% of their resources to contests over low value factors. As reported in table 2, both a non-parametric Wilcoxon signed-rank test and a t-test find this difference to be significant at the 1% level. The average allocation selected by a fixed matching pair over an entire experimental session is treated as a single observation in both tests. There were 4 sessions per valuation treatment and 10 fixed matching pairs per session, yielding a total of 40 observations.

6 Conclusions

Ex ante, one may expect resource allocations to approximate equilibrium predictions more closely under more responsive contest success functions, since competitors face stronger incentives to closely approximate a best response when contest success functions are more responsive to investment levels. Conversely, one may expect resource allocations to approximate equilibrium predictions more closely under less responsive contest success functions, since best responses to non-equilibrium resource allocations are farther from equilibrium predictions under more responsive contest success functions.

Accordingly, the experimental design of this study varies the responsiveness of the contest success function across treatment conditions. Equilibrium predictions were identical across treatments, but observed allocations were significantly closer to equilibrium under less responsive success functions. Observed behavior may have approximated equilibrium predictions more closely under less responsive success functions because best responses to non-equilibrium allocations are closer to equilibrium predictions under less responsive success functions. These results suggest that a careful consideration of non-equilibrium incentives provides useful information about the reliability of equilibrium predictions.

The present study investigates a particular class of conflicts where competitors allocate resources to compete for shares of complementary factors. However, additional research is needed to better understand how strategic features can influence the reliability of equilibrium predictions in other settings. The experimental design of the present study varies the responsiveness of the success function across treatment conditions, but it does not vary the number of competitors, the number of factors, or level of complementarity between factors. Further research is needed to investigate how these features influence allocation behavior.

A Proofs

Proof of Theorem 1. Suppose competitor i allocates zero resources to contest k such that $x_{ik} = 0$. Let $\hat{x}_i$ such that $\hat{x}_{ik} = \varepsilon \in (0,1)$ and $\hat{x}_{ib} = 1 - \varepsilon$. If $x_{ik} = 0$ and $x_{jk} \neq 0$ then $\pi_i(x) = 0 < \pi_i(\hat{x}_i, x_j)$. If $x_{ik} = x_{jk} = 0$ then taking the limit as $\varepsilon \to 0$ obtains.

$$\lim_{\varepsilon \to 0} \pi_i(\hat{x}_i, x_j) = \frac{\beta}{v_k + 2v_b} > \frac{\beta}{2} = \pi_i(x) \tag{4}$$

Hence $x_{ik} > 0$ in every Nash equilibrium. Let $g_i(x) = -\frac{\beta}{\pi_i(x)}$ so differentiating g_i with respect to x_{ik} yields

$$\frac{\partial g_i}{\partial x_{ik}} = \frac{\alpha v_k [1 - y_{ik}(x)]}{y_{ik}(x) x_{ik}} \tag{5}$$

Since $y_{ik}(x)$ is increasing in x_{ik}, the numerator of (5) is decreasing in x_{ik} and the denominator is increasing in x_{ik} so $\frac{\partial g_i^2}{\partial x_{ik}^2} < 0$. If $b \neq k$ then (5) is constant in x_{ib} so $\frac{\partial g_i^2}{\partial x_{ib}^2} = 0$. Hence g_i is strictly concave in x_i and π_i is strictly quasiconcave in x_i. The first order conditions on x_i for the maximization of π_i state that $\frac{\partial \pi_i}{\partial x_{ik}} = \frac{\partial \pi_i}{\partial x_{ib}}$ so we have

$$\frac{v_k [1 - y_{ik}(x)]}{y_{ik}(x) x_{ik}} = \frac{v_b [1 - y_{ib}(x)]}{y_{ib}(x) x_{ib}} \tag{6}$$

$$\frac{v_k y_{jk}(x)}{y_{ik}(x) x_{ik}} = \frac{v_b y_{jb}(x)}{y_{ib}(x) x_{ib}} \tag{7}$$

$$\frac{v_k y_{jk}(x) y_{ib}(x)}{v_b y_{jb}(x) y_{ik}(x)} = \frac{x_{ik}}{x_{ib}} \tag{8}$$

Since π_i is strictly quasiconcave in x_i and $x_{ik} > 0$ in every equilibrium, the first order conditions are necessary and sufficient for equilibrium so

$$\frac{x_{ik}}{x_{ib}} = \frac{x_{jk}}{x_{jb}} \tag{9}$$

$$\frac{x_{ik}}{x_{jk}} = \frac{x_{ib}}{x_{jb}} \tag{10}$$

$$\frac{x_{ik}^{\alpha}}{x_{ik}^{\alpha} + x_{jk}^{\alpha}} = \frac{x_{ib}^{\alpha}}{x_{ib}^{\alpha} + x_{jb}^{\alpha}} \tag{11}$$

$$y_{ik}(x) = y_{ib}(x) \tag{12}$$

Hence there exists $\bar{y}_i(x) = y_{ik}(x) = y_{ib}(x)$. Substituting this into equation

(8) yields

$$\frac{v_k \bar{y}_j\left(x\right) \bar{y}_i\left(x\right)}{v_b \bar{y}_j\left(x\right) \bar{y}_i\left(x\right)} = \frac{x_{ik}}{x_{ib}} \tag{13}$$

$$\frac{v_k}{v_b} = \frac{x_{ik}}{x_{ib}} \tag{14}$$

Since $x_{i1} + x_{i2} = v_1 + v_2 = 1$ we have $x_{ik} = v_k$. □

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