Continuous Feedback in School Choice Mechanisms*

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Abstract

Standard implementations of school choice mechanisms reveal assignments to participants only after all preference reports have been finalized. This paper experimentally investigates novel implementations that provide participants with assignment feedback throughout the preference reporting period. This feedback has no effect on the equilibrium predictions, but adaptive models predict that it will promote rational preference revelation by providing boundedly rational participants with increased opportunity for learning and adjustment. To test this hypothesis, the experiment implements both conventional discrete feedback and continuous assignment feedback in three widely employed school choice mechanisms: the deferred acceptance mechanism, the top trading cycles mechanism, and the Boston mechanism. In line with theoretical predictions from adaptive models, subjects achieved equilibrium assignments far more often under continuous assignment feedback, suggesting that policy makers can improve the effectiveness of school choice mechanisms by providing participants with more feedback throughout the preference reporting period.

*This research was supported by NSF grant SES-1458541. We would like to thank thank Alexander L. Brown, Catherine E. Eckel, Rodrigo A. Velez, and the experimental research team at Texas A&M for their continual support and feedback. We are also grateful for the insightful comments from attendees of the 2014 North American Meeting of the Economic Science Association, the the 2014 European Meeting of the Economic Science Association, and the 2016 Texas Economic Theory Camp.

1 Introduction

Children in the United States are traditionally assigned to public schools based on where they live, but a growing number of public school districts now allow parents to indicate their preferences over the schools their children attend. Since each school can support only a limited number of students, it is often impossible to give every student her most preferred school. To resolve these shortages, policy makers frequently employ student assignment mechanisms that assign each student to a school based on both the reported student preferences and the legally determined student priorities.

Under some of these mechanisms, participants have an incentive to strategically misreport their preferences. Some parent groups have even explicitly recommended particular misreporting strategies.[1] Misreported preferences can prevent policymakers from accurately evaluating mechanism efficiency and make it difficult to reliably achieve policy goals. To encourage truthful preference reports, mechanism designers typically recommend the use of strategy-proof assignment mechanisms under which participants never have an incentive to misreport their preferences. However, previous studies[2] find that even strategy-proof mechanisms fail to reliably induce truthful preference revelation from boundedly rational participants.

Standard implementations of school choice mechanisms only reveal assignments to participants at the end of the reporting period, after all preference reports have been finalized. This paper also considers novel implementations that provide participants with on-demand assignment feedback throughout the preference reporting period. This study is the first to experimentally investi-

[1] See Abdulkadiroglu et al. (2006) for more details regarding these misreporting strategies. [2] For example, Chen and Sönmez (2006) find that subjects misrepresent their preferences 50% of the time under a top trading cycles mechanism. Similarly, Pais and Pintér (2008) find that subjects misrepresent their preferences 33% of the time under a full information deferred acceptance mechanism.

gate such implementations. Adaptive models predict that this type of feedback can reduce confusion and promote rational preference revelation by providing boundedly rational participants with increased opportunity for learning and adjustment. To test this hypothesis, the experiment conducts both conventional discrete feedback and continuous assignment feedback implementations of three widely employed school choice mechanisms: the deferred acceptance mechanism, the top trading cycles mechanism, and the Boston mechanism.

The deferred acceptance mechanism and the top trading cycles mechanism are both strategy proof, so both have a Nash equilibrium in weakly dominant strategies under which participants truthfully report their preferences. Under the top-trading cycles mechanism, the dominant strategy Nash equilibrium always yields a Pareto optimal assignment. Under the deferred acceptance mechanism, it always yields an assignment that eliminates justified envy.[3] In contrast, the Boston mechanism is manipulable. It has no dominant strategy Nash equilibrium and it frequently gives participants an incentive to misreport their preferences. However, the set of Nash equilibrium assignments under the Boston mechanism coincide exactly with the set of assignments that eliminate justified envy as noted by Ergin and Sönmez (2006).

Advances in processor speed and network connectivity have largely eliminated the computational barriers to providing on-demand assignment feedback throughout the preference reporting period. Many school districts already provide online computerized web interfaces for preference reporting and limited forms of continuous feedback have already been employed by some school districts. Continuous feedback regarding the first choices of other participants in the Boston mechanism was provided by the Wake County Public School System (Dur et al., 2015). Inner Mongolia provided continuous feedback in a dynamic queuing mechanism where subjects only report their first choices (Gong and Liang, 2016). However, it is now computationally feasible to pro-

[3] See Subsection 2.1 for details regarding the formal definition of justified envy.

vide participants with on-demand information regarding their school assignment at any time throughout the preference reporting period. The experiment reported by this paper investigates how this type of assignment feedback might effect preference reporting behavior.

This study connects two distinct strands of literature: the school choice literature in mechanism design theory, and experimental literature investigating dynamic behavior in continuous-time games. The school choice mechanism design literature provides an axiomatic analysis of rational preference revelation behavior under school choice mechanisms. Abdulkadiroglu and Sönmez (2003) describe the school choice problem and discuss the fundamental tradeoff between Pareto efficiency and the elimination of justified envy. They also describe a variation of the top trading cycles mechanism introduced by Shapley and Scarf (1974) which we investigate in this study. A powerful characterization of the Nash equilibria of the Boston mechanism was provided by Ergin and Sönmez (2006) and the student optimal deferred acceptance mechanism was described by Gale and Shapley (1962).

Previous experimental studies, such as Chen and Sönmez (2006), conducted school choice mechanisms in discrete periods, which is ideal for the study of static one-shot mechanisms. In contrast, continuous-time experimental studies have successfully investigated dynamic behavior in a variety strategic settings involving continuous-time interaction. For example, Cason et al. (2013) conduct a experimental investigation of dynamic behavior in continuous-time rock-paper-scissors games and Oprea et al. (2011) conduct a continuous-time experimental study of evolutionary dynamics in the Hawk-Dove game. Both studies provide subjects with continuous feedback and allow subjects to adjust their strategies asynchronously. This paper employs continuous time experimental methodology to investigate dynamic preference revelation behavior in three widely employed school choice mechanisms.

In contrast to the predictions of dominant strategy Nash equilibrium but in line with theoretical predictions from the adaptive best response model, Subjects in all three school choice mechanisms were significantly more likely to achieve equilibrium assignments under continuous assignment feedback implementations than under conventional discrete feedback implementations. Subjects in the top trading cycles mechanism were significantly more likely to receive their most preferred school under continuous assignment feedback than under conventional discrete feedback. Subjects in the deferred acceptance mechanism and the Boston mechanism, were significantly less likely to face justified envy under continuous assignment feedback than under conventional discrete feedback. These results suggest that the provision of continuous assignment feedback in school choice mechanisms can provide participants with greater opportunity for learning and adjustment, producing more rational preference reports and helping policy makers to more reliably achieve their goals.

The remainder of this paper is organized as follows. Section 2 describes the school choice mechanisms under consideration and their respective theoretical properties. Section 3 provides a description of the experimental design and procedures. Section 4 covers the hypothesis that are tested. Section 5 presents the results and Section 6 concludes.

2 Theory

2.1 The School Choice Environment

This study experimentally investigates school choice environments illustrating a fundamental tradeoff between Pareto efficiency and the elimination of justified envy.[4] Each of three schools can accept up to n students and each student can be assigned to only one school. Each student has strict preferences over schools and each school has a strict priority rankings over students. There are three types of students and there are n students of each type. Stu-

[4] A similar environment with only one student of each type is discussed by Abdulkadiroglu and Sönmez (2003) and Roth (1982).

dent preferences over schools are given by

Student	1	2	3
	b	a	a
Preference	a	b	b
	c	c	c

where higher vertical position indicates a higher preference ranking, so type 1 students prefer school b over school a and prefer school a over school c. Similarly, school priority rankings over students satisfy

School	a	b	c
	1	2	2
Priority	3	1	1
	2	3	3

so school a prefers type 1 students over type 3 students and type 3 students over type 2 students. A student x is said to have *justified envy* towards a student y if student x prefers the school assigned to y and student x is also ranked higher than y at this school. If no student has justified envy under a particular assignment of students to schools we say that this assignment eliminates justified envy. In general, several distinct assignments may eliminate justified envy in a given school choice environment. However, in these particular environments, the only assignment that eliminates justified envy is given by

$$\mu = \begin{pmatrix} 1 & 2 & 3 \\ a & b & c \end{pmatrix}$$

where all type 1 students are assigned to school a, all type 2 students are assigned to school b, and all type 3 students are assigned to school c. These environments illustrate a fundamental tradeoff between Pareto efficiency and the elimination of justified envy because the assignment μ that uniquely eliminates justified envy is Pareto dominated by the assignment

$$\lambda = \begin{pmatrix} 1 & 2 & 3 \\ b & a & c \end{pmatrix}$$

where all type 1 students are assigned to school b, all type 2 students are assigned to school a, and all type 3 students are assigned to school c. The assignment λ Pareto dominates the assignment μ because type 1 students and type 2 student both prefer the schools they receive under λ to the schools they receive under μ while type 3 students receive the same school under both λ and μ. However, it should be noted that the Pareto optimal assignment λ fails to fully eliminate justified envy because it gives type 3 students justified envy towards type 2 students. Student 3 would prefer school a over school c, and school a ranks student 3 higher than student 2. Since the unique assignment μ that eliminates justified envy is Pareto dominated by λ, no Pareto optimal assignment can eliminate justified envy in this environment. Further, no student assignment mechanism can guarantee both the elimination justified envy and Pareto optimality. Policy makers generally face a fundamental tradeoff between Pareto efficiency and the elimination of justified envy in school choice environments.

2.2 Student Assignment Mechanisms

Student assignment mechanisms select an assignment of students to schools based on the priority rankings of each school and the preferences reported by students. Since it is impossible for any student assignment mechanism to ensure both Pareto optimality and the elimination of justified envy, the optimal student assignment mechanism for a particular school district depends partly on the particular goals of the individual policy maker. Hence different school districts may reasonably employ different student assignment mechanisms if they have different policy goals. In particular, this paper considers three widely employed assignment mechanisms: the Boston mechanism, the top trading cycles mechanism, and the student optimal deferred acceptance mechanism.

A student x is said to have justified envy towards a student y if the student x prefers the school s that is assigned to student y and the student x also has higher priority at school s than student y. If no student has justified envy under a particular assignment then the assignment is said to eliminate justified envy. A student assignment mechanism is said to eliminate justified envy if it always selects an assignment that eliminates justified envy under the reported preferences. Similarly, a mechanism is Pareto optimal if it always selects an assignment that is Pareto optimal under the reported preferences. A mechanism is said to be strategy proof if no student can ever benefit by unilaterally misreporting her preferences.

2.2.1 The Boston Mechanism

Under the Boston mechanism, each student initially applies to her top choice of schools according to her reported preferences. Each school accepts applicants in priority order until it runs out of seats. The remaining students apply to their second choice of schools according to their reported preferences. Again, each school accepts students in priority order until it runs out of seats. This process repeats until every student is assigned to a school. If students truthfully report their preferences, the Boston mechanism selects a Pareto optimal assignment, but the Boston mechanism is not strategy proof. Students can often benefit by misreporting their preferences. Ergin and Sönmez (2006) show that the set of Nash equilibrium assignments under the Boston Mechanism exactly coincide with the set of matchings that eliminate justified envy under the true preferences. However, as noted in section Subsection 2.1, these equilibrium assignments may be Pareto dominated.

In the experimentally implemented school choice environment, the unique assignment that eliminates justified envy is also the unique Nash equilibrium assignment for the Boston mechanism. Under this assignment, none of the students receive their most preferred school. It is Pareto dominated by the the Pareto optimal assignment under which all type 1 students and all type

2 students receive their most preferred school. In contrast to the deferred acceptance and top trading cycles mechanisms, the Boston mechanisms is manipulable, so it has no dominant strategy Nash equilibrium under which students accurately report their preferences. Further, in the Boston mechanism, equilibrium assignments can often can often only be obtained if participants misreport their preferences.

2.2.2 The Deferred Acceptance Mechanism

Under the student optimal deferred acceptance mechanism, each student initially applies to her top choice of schools according to her reported preferences. Each school tentatively accepts applicants in priority order until it runs out of seats. The remaining applications are rejected. Students whose applications were rejected then apply to their next highest choice of schools. Next, each school considers its new applicants along side those it has already tentatively accepted. It tentatively accepts its top priority students among this group until it run out of seats and rejects the remaining students. This process repeats until every student is assigned to a school.

Unlike the Boston mechanism, the deferred acceptance mechanism is strategy proof, so students never have an incentive to misreport their preferences. When students truthfully report their preferences, the deferred acceptance mechanism always selects an assignment that eliminates justified envy under the true preferences. Yet even when students report their preferences truthfully, the deferred acceptance mechanism may select a Pareto dominated assignment. In the school choice environment implemented by the experiment, the deferred acceptance mechanism selects the sole assignment μ that eliminates justified envy when students report their preferences truthfully. This assignment is Pareto dominated by the the Pareto optimal assignment λ where type 1 students and type 2 students receive their most preferred schools. None of students receive their most preferred school under the dominant strategy Nash equilibrium assignment μ of the student optimal deferred acceptance mechanism.

2.2.3 The Top Trading Cycles Mechanism

The top trading cycles mechanism constructs a directed graph based the priorities and reported preferences. Each school points to it's highest priority student and each student points to her most preferred school according to her reported preferences. Since there are a finite number of schools and students, the directed graph will have at least one cycle. Students who are part of a cycle are assigned to the school they point. Each of the remaining students point to their most preferred school according to their reported preferences among those schools that still have open seats. Each school points to their highest priority student among those students that remain unassigned. Students who are part of a cycle are assigned to the school they point to. This process repeats until every student is assigned to a school.

The top trading cycles mechanism strategy proof, so students have no incentive to misreport their preferences. However, unlike the deferred acceptance mechanism, the top trading cycles mechanism always selects a Pareto optimal assignment when students truthfully report their preferences. However, even if students truthfully report their preferences, the top trading cycles mechanism may select an assignment that fails to eliminate justified envy. In the school choice environment implemented by the experiment, the Pareto optimal assignment λ is the dominant strategy Nash equilibrium assignment for the top trading cycles mechanism. Under this assignment, type 1 students and type 2 students receive their most preferred school, but Type 3 students receive their least preferred school. Hence two thirds of the student population receive their most preferred school under this assignment, but it fails to eliminate justified envy since it gives type 3 students justified envy towards type 1 students.[5]

[5] See Subsection 2.1 for details regarding the justified envy of type 3 students towards type 1 students under the Pareto optimal assignment λ.

2.3 Adaptive Dynamics

Previous experimental studies[6] find that even strategy-proof mechanisms fail to reliably induce rational preference revelation if they wait to provide assignment feedback until after preference reports have been finalized. These failures to induce rational preference revelation can prevent strategy proof mechanisms from achieving equilibrium outcomes. They may result from bounded rationality, confusion, or disbelief regarding the incentives presented by strategy proof mechanisms. To ameliorate these problems, this study considers the implementation of school choice mechanisms that provide assignment feedback throughout the preference reporting period.

The provision of continuous feedback has no effect on the dominant strategy Nash equilibrium of these school choice mechanisms. Assignments remain exclusively determined by the preference reports selected at the end of the reporting period. However, by allowing for adaptive learning and adjustment, the provision of assignment feedback throughout the preference reporting period may induce boundedly rational agents to exhibit more rational preference revelation behavior, helping school choice mechanisms to achieve equilibrium assignments. Adaptive models describe the adjustment behavior of boundedly rational agents in environments with informational feedback, such as school choice mechanisms with assignment feedback. One such model, the best response dynamic considered by Gilboa and Matsui (1991) and Matsui (1992), describes agents who myopically switch to best responses. This adaptive dynamic is also closely related to the fictitious play dynamic discussed by Brown (1951). Formally, under the best response dynamic, if agents are sending the report profile r then the probability that agent i will switch from preference

[6] For example, Chen and Sönmez (2006) find that subjects misrepresent their preferences 50% of the time under a top trading cycles mechanism with discrete feedback. Similarly, Pais and Pintér (2008) find that subjects misrepresent their preferences 33% of the time under a deferred acceptance mechanism with discrete feedback.

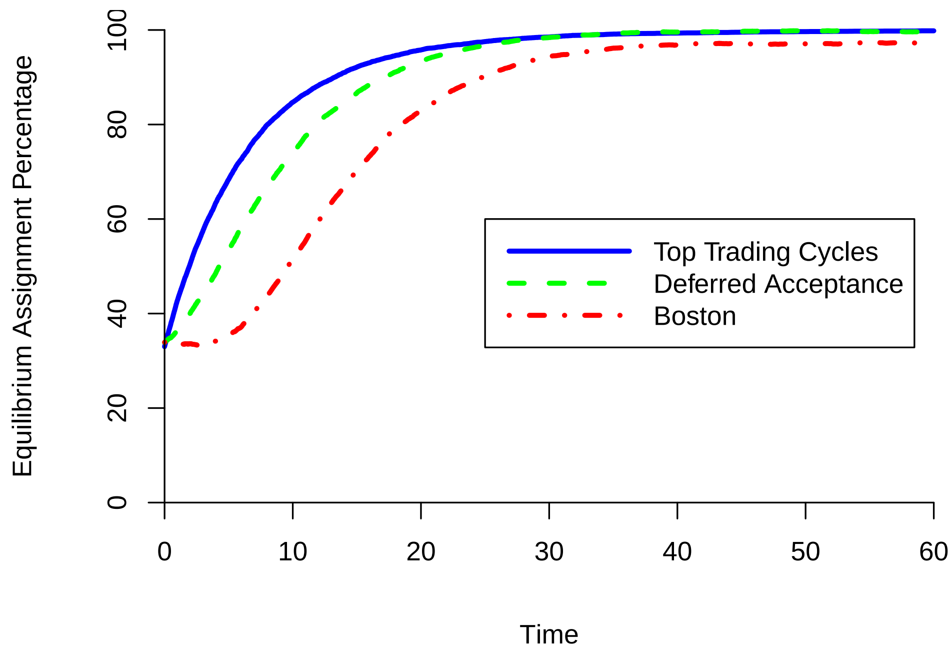

Figure 1: Equilibrium Assignment Percentages under the Best Response Dynamic

report r to the preference report q is given by

$$P\left(q_i|r\right) = \frac{f\left(q_i|r\right)}{\sum\limits_{x_i \in R_i} f\left(x_i|r\right)} \qquad \text{where}$$

$$f\left(q_i|r\right) = \begin{cases} 1 & \text{if } q_i \in \underset{x_i}{\operatorname{argmax}}\ \pi_i\left(x_i, r_{-i}\right) \\ 0 & \text{otherwise} \end{cases}$$

Figure 1 depicts theoretical predictions from the best response dynamic regarding equilibrium assignment percentages in school choice mechanisms with continuous assignment feedback. The horizontal axis denotes time over the course of a reporting period and the vertical axis denotes the percentage of participants receiving their equilibrium assignment. Each line depicts the mean path of a particular school choice mechanism under the best response

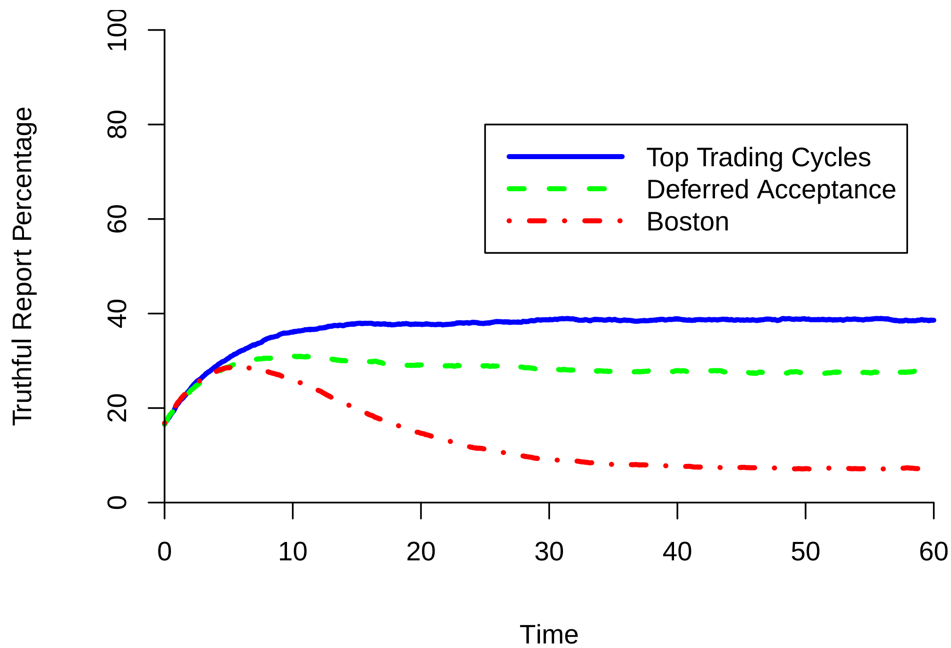

Figure 2: Equilibrium Assignment Percentages under the Best Response Dynamic

dynamic in the experimentally investigated school choice environment.[7] Note that both the deferred acceptance mechanism and the top trading cycles mechanism rapidly converge to equilibrium assignments, while the Boston mechanism converges slowly and exhibits persistent deviation from equilibrium, illustrating how the manipulability of the Boston mechanism can produce to dynamic instability even under continuous feedback.

Figure 2 depicts the truthful reporting percentages predicted by the best response dynamic in school choice mechanisms with continuous assignment feedback. The horizontal axis denotes time over the course of a reporting period and the vertical axis denotes the percentage of participants who report their preferences truthfully. Each line depicts the mean path of a school choice mechanism under the best response dynamic in the experimentally investigated school choice environment. Note that the strategy proof top trading cycles and

[7] See Subsection 2.1 for details regarding the school choice environment under consideration

	Top Trading Cycles	Deferred Acceptance	Boston
Discrete Feedback	3 Sessions	3 Sessions	3 Sessions
Continuous Feedback	3 Sessions	3 Sessions	3 Sessions

Table 1: 3x2 Experimental Design with Three Sessions Per Block

deferred acceptance mechanisms induce more truthful preference reports than the manipulable Boston mechanism, but none of these mechanisms reliably induce truthful preference reports under the best response dynamic.

The best response dynamic predicts that untruthful preference reports will persist in these strategy proof mechanisms because truth telling is only a weakly dominant strategy. Participants can often deviate from a truthful preference report to an untruthful preference report without affecting any particiapnt's assignment. Accordingly, the best response dynamic predicts that the provision of continuous assignment feedback can help boundedly rational agents to achieve equilibrium assignments in strategy proof mechanisms, but is unlikely to reliably induce truthful preference revelation.

3 Experimental Design and Procedures

This study implements a 2x3 experimental design with six experimental treatments illustrated by Table 1. Each column of this table denotes one of three widely employed school choice mechanisms: the deferred acceptance mechanism, the top trading cycles mechanism, and the Boston mechanism. For each of these mechanisms, this study implements an experimental treatment condition with continuous assignment feedback and another experimental treatment condition with standard discrete feedback. A total of eighteen experimental sessions were conducted, three for each of the six treatment blocks. Each experimental session was conducted with twenty-four subjects for a total

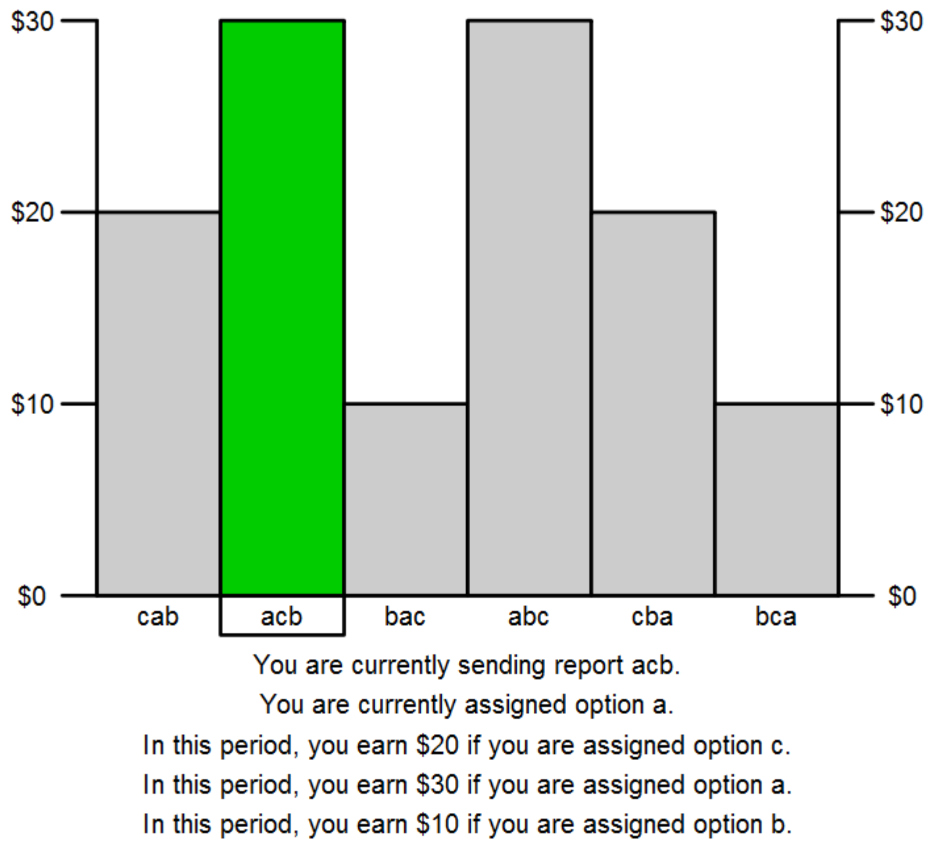

Figure 3: Experimental Interface under Continuous Feedback

four hundred and thirty two experimental subjects. Each subject participated in only one experimental session. All sessions were conducted at the Texas A&M Economic Research Laboratory.

During each experimental session, subjects were divided into three groups of eight participants. Members of each group were assigned one of the three student types described in Subsection 2.1. Each experimental session consisted of twelve reporting periods, each reporting period lasting for exactly one minute. At the beginning of each reporting period, subjects were informed about the earnings that they could receive from being assigned each of the three options: a, b, or c. This information remained visible to subjects

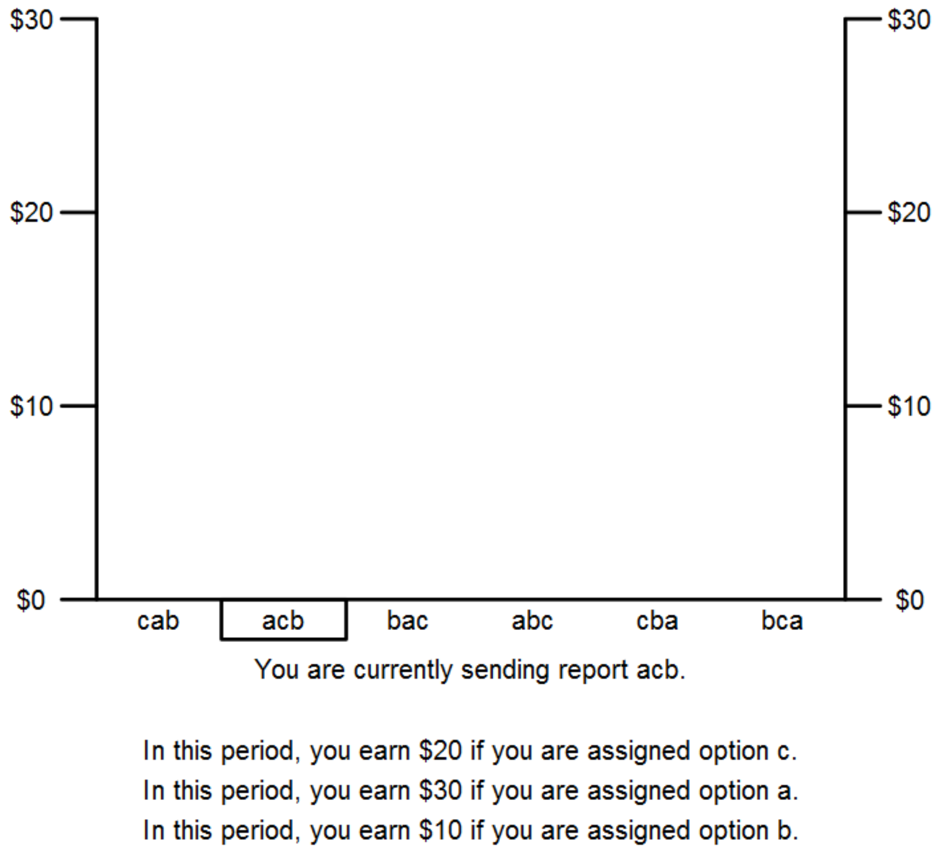

Figure 4: Experimental Interface under Discrete Feedback

for the duration of the experimental session. To avoid the possibility of introducing any psychological ordering or labeling bias, the labeling for each school and the order in which the options were presented was randomly reassigned at the beginning of each period.

Throughout each reporting period, subjects were free to adjust their preference reports as frequently as desired. At the end of each reporting period, all preference reports were finalized and assignments were made based on these finalized preference reports. Figure 3 depicts the experimental interface during the preference reporting period under continuous assignment feedback and Figure 4 illustrates the experimental interface during the preference reporting period under conventional discrete feedback. Under the discrete feedback treatment, subjects could observe their assignments at the end of each reporting period, after all preference reports were finalized. Under the continuous feedback treatment, subjects could also observe their tentative assignments under the currently selected preference reports throughout the one minute reporting period. Under both treatment conditions, subjects could observe their assignment at the end of the preference reporting period. At the end of each session, subjects were paid the average of their earnings over all periods plus a five dollar participation bonus.

4 Hypotheses

The dominant strategy Nash equilibria of the school choice mechanisms under consideration are unaffected by the provision of continuous assignment feedback. In contrast, the adaptive best response dynamic predicts that continuous assignment feedback will help boundedly rational agents achieve equilibrium assignments by providing more opportunity for learning and adjustment. Asynchronous adjustment to myopic best responses produces convergence[8] to equilibrium assignments, but this process of myopic adjustment can only occur when participants receive assignment feedback during the preference reporting period. Accordingly, the provision of continuous assignment feedback throughout the preference reporting period is hypothesized to significantly increase the proportion of equilibrium assignments under all three school choice mechanisms.

Hypothesis 1. *School choice mechanisms will achieve equilibrium assignments more often when they provide assignment feedback throughout the preference reporting period than when they only provide assignment feedback after*

[8] See Subsection 2.3 for more details regarding the theoretical predictions of the best response dynamic.

all preference reports have been finalized.

The top trading cycles mechanism is strategy proof and it always achieves Pareto efficiency in the dominant strategy Nash equilibrium. In the experimentally implemented school choice environment,[9] the dominant strategy Nash equilibrium of the top trading cycles mechanism yields an assignment under which all students of type 1 and all students of type 2 are assigned to their most preferred school. However, school a is the favorite of both type 2 students and type 3 students, so it is not possible to assign more than two thirds of the student population to their most preferred school. Thus, if the top trading cycles mechanism makes dominant strategy Nash equilibrium assignments, then it will maximize the number of participants that are assigned to their favorite school. This theoretical prediction motivates the second hypothesis.

Hypothesis 2. *The top trading cycles mechanism will assign more students their most preferred school when it provides continuous feedback than when it only provides receive discrete feedback.*

The student optimal deferred acceptance mechanism is strategy proof and always eliminates justified envy in the dominant strategy Nash equilibrium. In the experimentally implemented school choice environment, its dominant strategy Nash equilibrium yields the unique assignment which completely eliminates justified envy under the true preferences in this school choice environment.[10] Thus, if the student optimal deferred acceptance mechanism makes dominant strategy Nash equilibrium assignments, then it will eliminate justified envy. This theoretical prediction motivates the third hypothesis.

Hypothesis 3. *The deferred acceptance mechanism will eliminate more justified envy when it provides continuous feedback than when it only provides discrete feedback.*

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[9] See Subsection 2.1 for details regarding the school choice environment.

[10] See Subsection 2.1 for details regarding the unique assignment that eliminates justified envy in this environment.

The Boston mechanism is not strategy proof but its set of Nash equilibrium outcomes is always equal to the set of assignments that eliminate justified envy. In the experimentally implemented school choice environment, its unique Nash equilibrium yields the unique assignment which completely eliminates justified envy under the true preferences in this school choice environment. Thus, if the Boston mechanism makes Nash equilibrium assignments, then it will eliminate justified envy. This theoretical prediction motivates the fourth hypothesis.

Hypothesis 4. *The Boston mechanism will eliminate more justified envy when it provides continuous feedback than when it only provides discrete feedback.*

No school choice mechanism can guarantee both Pareto efficiency and the elimination of Justified envy in every school choice environment. In the experimentally investigated school choice environment, the unique assignment that eliminates justified envy is not Pareto optimal and it does not give any of the students their most preferred school, so we not hypothesize that continuous assignment feedback will increase the number of students who receive their favorite schools in the deferred acceptance mechanism or the Boston mechanism. Conversely, the Pareto optimal assignment selected by the dominant strategy Nash equilibrium of the top trading cycles mechanism does not eliminate justified envy, so we do not hypothesize that continuous assignment feedback will increase the elimination of justified envy in the top trading cycles mechanism. Further, while adaptive models predict that continuous assignment feedback will help participants achieve equilibrium assignments, they do not predict that continuous assignment feedback will induce truthful preference reports from boundedly rational agents.[11] Consequently, we do not hypothesize that that continuous feedback will increase the proportion of truthful preference

[11] See Subsection 2.3 for details regarding the predictions from adaptive models regarding truthful preference revelation under continuous assignment feedback.

reports.

5 Results

Figure 5 illustrates the proportion of equilibrium assignments under each of the six experimental treatments. The three mechanisms under consideration, are listed along the horizontal axis. For each of these mechanisms, the height of the left-hand bar denotes the percentage of equilibrium assignments under discrete feedback and the height of the right-hand bar denotes the percentage of equilibrium assignments under continuous feedback. The vertical axis denotes the percentage of subjects who received their dominant strategy equilibrium assignment. Error bars indicate 95% confidence intervals on the percentage of equilibrium assignments.

Table 2 presents hypothesis tests for the effect of continuous feedback on equilibrium assignments. A non-parametric Mann-Whitney-Wilcoxon rank-sum test finds that equilibrium assignment percentages are significantly higher under continuous feedback at the one percent level. Further, a t-test finds that all three mechanisms separately achieve equilibrium assignments significantly more often under continuous feedback than under discrete feedback at the one percent level. Although the provision of continuous feedback has no effect on the dominant strategy Nash equilibrium predictions, the adaptive best response dynamic anticipates this result.[12] This result indicates that the provision of continuous feedback may induce boundedly rational agents to exhibit more rational preference revelation behavior by allowing learning and adjustment during the preference reporting period.

Result 1. *All three school choice mechanisms achieved equilibrium assignments significantly more often when they provided subjects with continuous*

[12] See Subsection 2.3 for more details regarding the predictions of the best response dynamic.

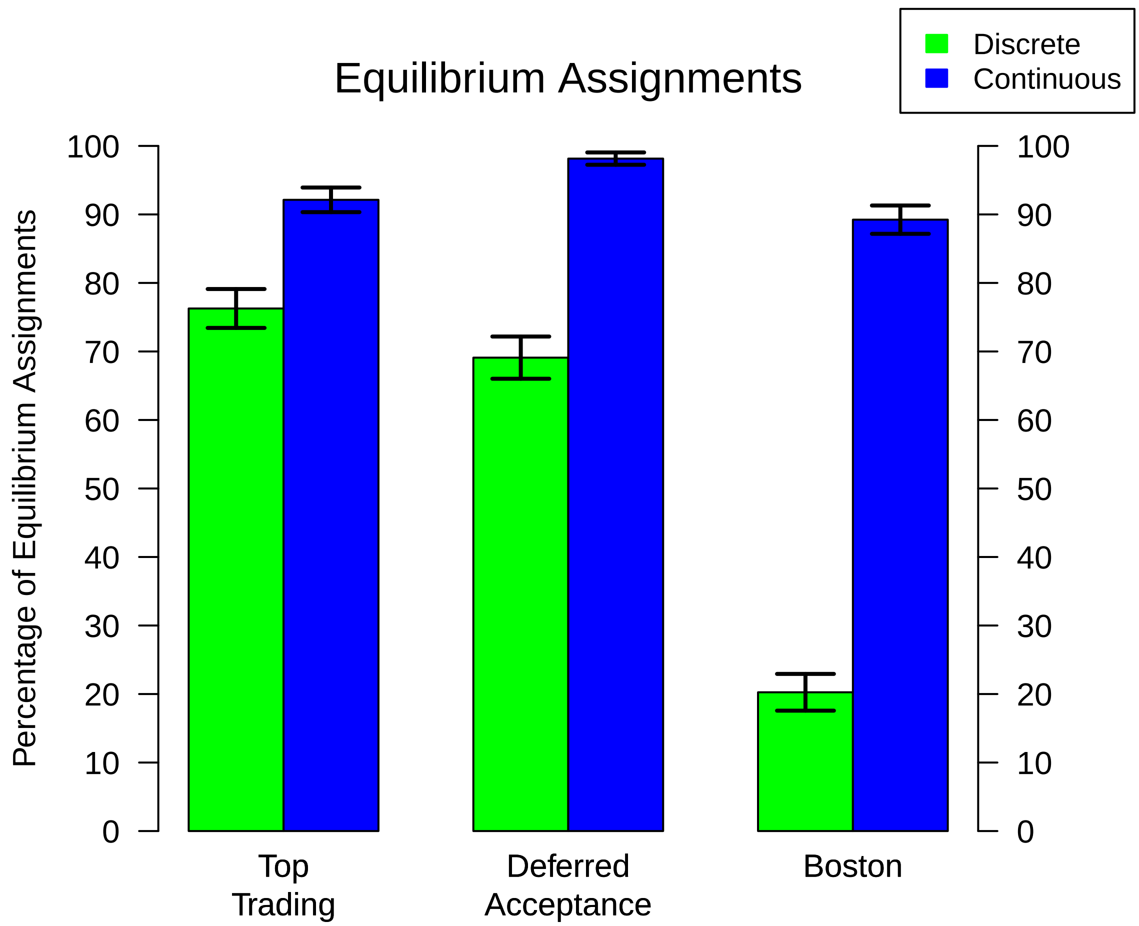

Figure 5: Proportion of Equilibrium Assignments by Treatment

	Feedback		t-test	Rank-Sum Test
	Discrete	Continuous	p-value	p-value
Top Trading Cycles	0.762	0.921	<0.001	
Deferred Acceptance	0.690	0.981	<0.001	<0.001
Boston	0.202	0.892	<0.001	

Table 2: Means and hypothesis tests regarding the proportion of equilibrium assignments. The unit of observation for the t-tests is one period. The unit of observation for the Mann-Whitney-Wilcoxon rank-sum test is one session.

feedback than when they provided subjects with discrete feedback.

Figure 6 illustrates the empirical assignment dynamics over time within the preference reporting period. The horizontal axis of each graph indicates seconds within a preference reporting period. The vertical axis of each graph indicates the average percentage of participants receiving their equilibrium assignment. The first graph illustrated the empirical results under the student optimal deferred acceptance mechanisms. The second graph illustrates the empirical results under the Boston mechanism. The third graph illustrates the empirical results under the top trading cycles mechanism. In all three graphs, the blue line illustrates the empirical path of the equilibrium assignment percentage under continuous feedback throughout the reporting period. The green line illustrates the empirical path of the equilibrium assignment percentage under conventional discrete feedback during each reporting period. Under all treatment conditions, assignments were made exclusively based on the preference reports submitted at the end of the reporting period. Until the end of the preference reporting period, assignments were tentative and subject to change.

Even under discrete feedback, many subjects in the strategy-proof mechanisms still adjusted their preference reports so as to obtain their equilibrium assignments. However, under discrete-feedback implementations of the manipulable Boston mechanism, the percentage of equilibrium assignments decreased over the preference reporting period. The Nash equilibrium preference reports for the Boston mechanism are generally untruthful, yet under discrete feedback, about two-thirds[13] of the subject populations naively selected truthful preference reports, suggesting that discrete-feedback was insufficient for many subjects to develop strategic preference reports. In contrast, under continuous assignment feedback, only about one-fifth of the subjects in the

[13] See Figure 9 for more details regarding the empirical dynamics of truthful reporting.

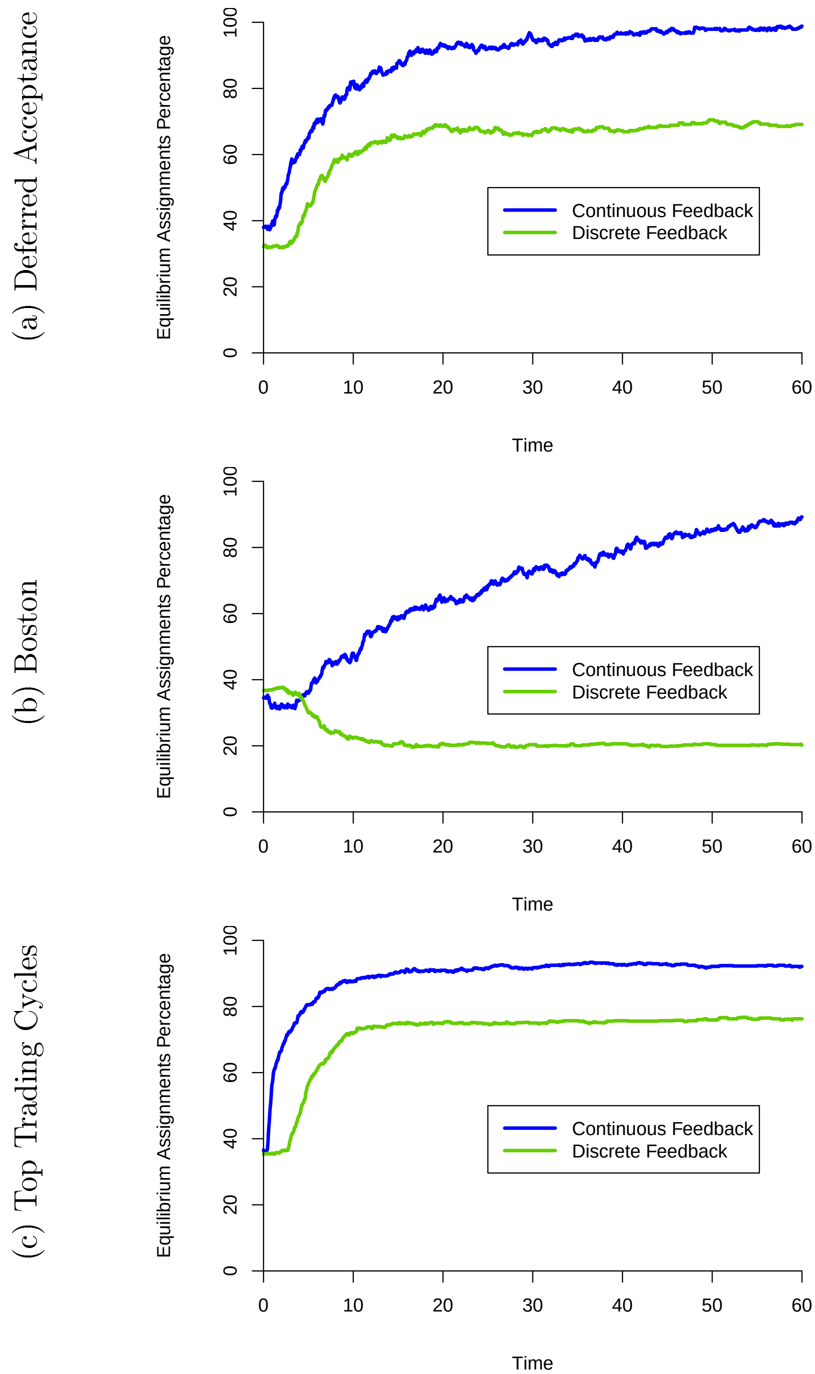

Figure 6: Percentage of equilibrium assignments over the preference reporting period. The first graph illustrated the empirical results under the student optimal deferred acceptance mechanisms. The second graph illustrates the empirical results under the Boston mechanism. The third graph illustrates the empirical results under the top trading cycles mechanism.

Boston mechanism selected truthful preference reports and the the equilibrium assignment percentage increased over the preference reporting period, suggesting that continuous-feedback helped subjects to develop strategic preference reports. Under all three school choice mechanisms, the percentage of equilibrium assignments at the end of the reporting period was higher under continuous feedback than under discrete feedback, indicating that subjects selected more rational preference reports when they had access to assignment feedback throughout the preference reporting period.

Figure 7 illustrates the elimination of justified envy under each of the six treatments. The vertical axis denotes the percentage of subjects who had no justified envy towards others under their true preferences.[14] The three mechanisms under consideration, are listed along the horizontal axis. For each of these mechanisms, the height of the left-hand bar denotes the elimination of justified envy under discrete feedback and the height of the right-hand bar denotes the elimination of justified envy under continuous feedback. Error bars indicate 95% confidence intervals on the percentage of students without justified envy.

Table 3 presents hypothesis tests for the effect of continuous feedback on the elimination of justified envy. A t-test finds that all three of these school choice mechanisms eliminate significantly more justified envy under continuous feedback than under discrete feedback at the one percent level. An F-test test rejects the null hypothesis that mechanisms were equally likely to eliminate justified under continuous feedback and discrete feedback at the one percent level. As discussed in Subsection 2.2, the assignment that uniquely eliminates justified envy in this environment is the equilibrium outcome for both the deferred acceptance mechanism and the Boston mechanism. Hence this result is consistent with the increase in equilibrium assignments from continuous feedback under these two mechanisms.

[14] A formal definition for the concept of justified envy can be found in Subsection 2.2

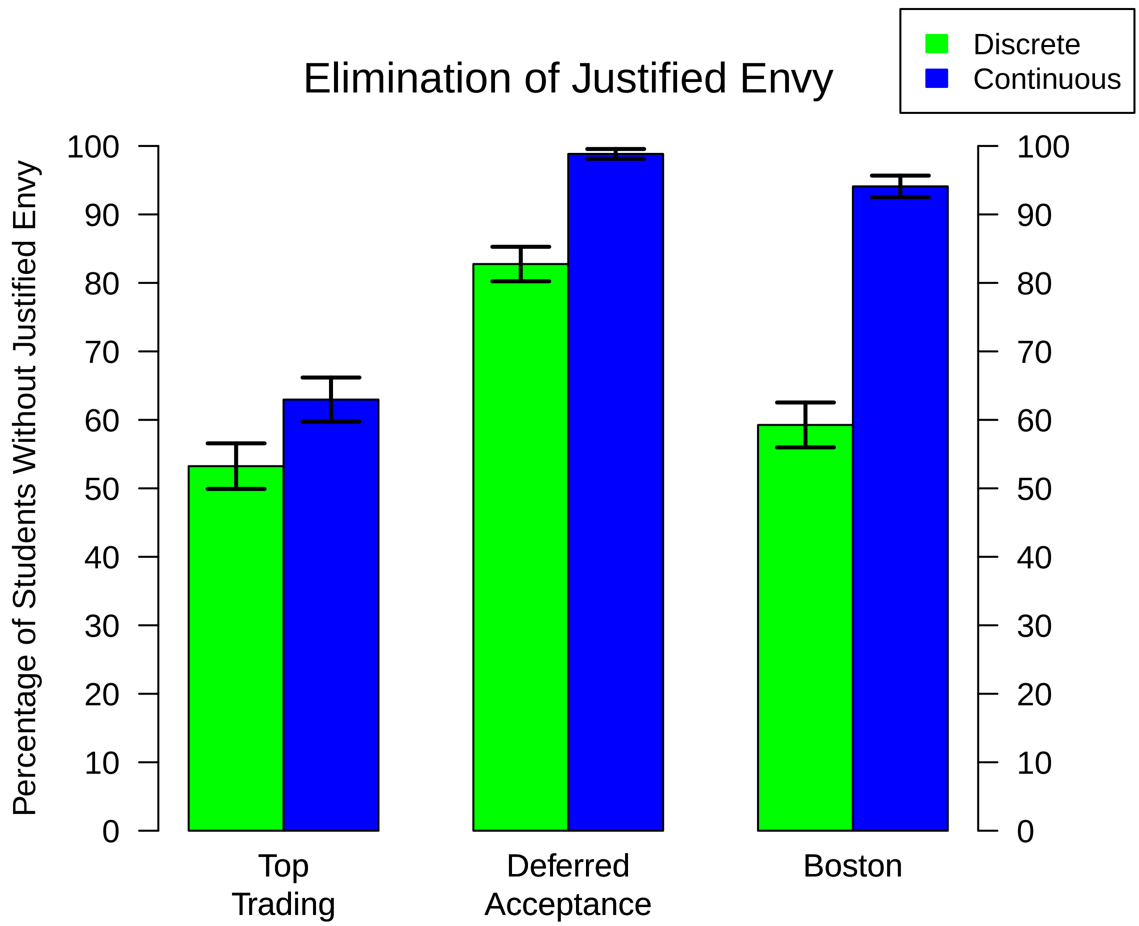

Figure 7: Elimination of Justified Envy by Treatment

	Feedback		t-test	F-Test
	Discrete	Continuous	p-value	p-value
Top Trading Cycles	0.53241	0.62963	<0.001	
Deferred Acceptance	0.82755	0.98843	<0.001	<0.001
Boston	0.59259	0.94097	<0.001	

Table 3: Hypothesis tests regarding the elimination of justified envy. The unit of observation is one period.

Result 2. *All three school choice mechanisms eliminated significantly more justified envy when they provided subjects with continuous feedback than when they provided subjects with conventional discrete feedback.*

The dominant strategy Nash equilibrium outcome of the top trading cycles mechanism[15] does not eliminate justified envy because it gives type 3 students justified envy towards type 2 students. However, it keeps type 1 and type 2 students free from justified envy as they receive their most preferred school. Consistent with this theoretical prediction, about two-thirds of subjects received their most preferred school under the top trading cycles mechanism with continuous feedback. In contrast, under conventional discrete feedback, only 53% of subjects were free from justified envy, far less than predicted by the dominant strategy Nash equilibrium. Since such a large amount of justified envy occurred under the top trading cycles discrete feedback treatment, better convergence to the dominant strategy equilibrium assignment lead to the elimination of more justified envy under the continuous feedback top trading cycles treatment.

Figure 8 illustrates the proportion of subjects who received their most preferred assignment under each of the six experimental treatment conditions. The three mechanisms under consideration, are listed along the horizontal axis. For each of these mechanisms, the height of the left-hand bar denotes the percentage of most preferred assignments under under discrete feedback and the height of the right-hand bar denotes the percentage of most preferred assignments under continuous feedback. The vertical axis denotes the percentage of subjects who who were assigned their most preferred option. Since school a is the favorite of both type 2 and type 3 students, it is not possible to assign more than two thirds of the student population to their most preferred school. Error bars indicate 95% confidence intervals on the percentage

[15] See subsubsection 2.2.3 for more details on the dominant strategy Nash equilibrium of the top trading cycles mechanism.

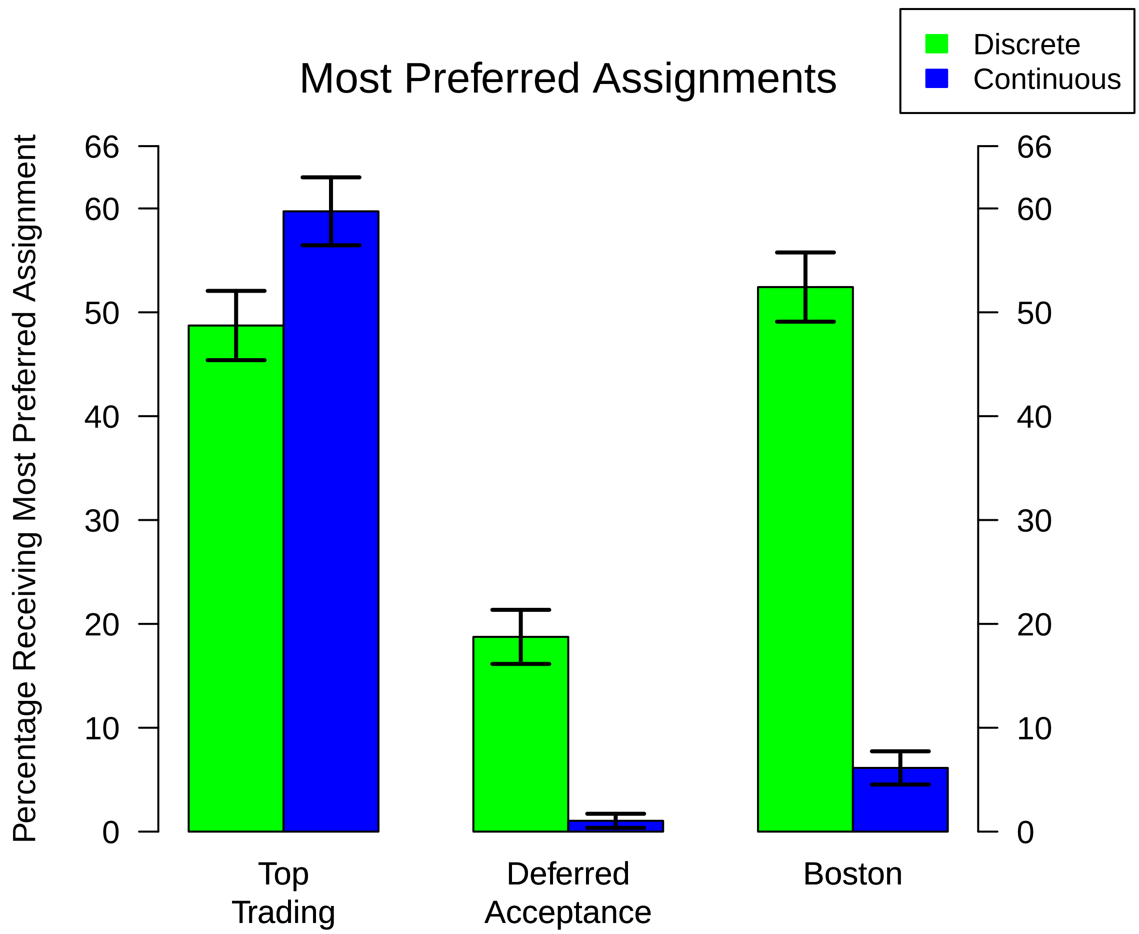

Figure 8: Proportion of Most Preferred Options by Treatment

	Feedback		t-test	F-Test
	Discrete	Continuous	p-value	p-value
Top Trading Cycles	0.48727	0.59722	<0.001	
Deferred Acceptance	0.18750	0.01042	<0.001	<0.001
Boston	0.52431	0.06134	<0.001	

Table 4: Hypothesis tests regarding the proportion most preferred assignments. The unit of observation is one period.

of students receiving their most preferred assignments.

Table 4 presents hypothesis tests for the effect of continuous feedback on the percentage of students receiving their most preferred assignment. A t-test finds that the top trading cycles mechanism assigned subjects their most preferred option significantly more often under continuous feedback than under discreet feedback at the one percent level. An F-test rejects the joint hypotheses of equal most preferred assignments across feedback treatments at the one percent level. These results are consistent with the increase in equilibrium assignments under continuous feedback since the top trading cycles mechanism maximizes the proportion of participants who receive their most preferred option in the dominant strategy Nash equilibrium of the experimentally investigated school choice environment.

Result 3. *The top trading cycles mechanism gave subjects their most preferred option significantly more often when it provided subjects with continuous feedback than when it provided subjects with discrete feedback.*

School choice mechanisms face a fundamental tradeoff between efficiency and the elimination of justified envy, so eliminating justified envy can necessitate inefficient outcomes. The dominant strategy Nash equilibrium of the student optimal deferred acceptance mechanism and the Nash equilibrium of the Boston mechanism in this school choice environment[16] both select the unique assignment that fully eliminates justified envy but does not assign any of the participants to their most preferred school. Even under discrete feedback, the strategy-proof deferred acceptance mechanism achieved over two-thirds[17] equilibrium assignments. Consequently, it rarely assigned students to their most preferred school.

In contrast, discrete feedback implementations of the manipulable Boston

[16] See Subsection 2.1 for more details regarding the experimentally investigated school choice environment. [17] See Figure 6 for more details on the empirical convergence dynamics.

mechanism only made about one-fifth equilibrium assignments and over half[18] of the students in these treatments naively selected truthful preference reports. Consequently, over half of subjects in discrete-feedback implementations of the Boston mechanism were assigned to their most preferred school, since the Boston mechanism selects a Pareto efficient assignment when subjects naively send truthful preference reports. All three school choice mechanisms were significantly more likely to make equilibrium assignments under the continuous feedback treatment, so the deferred acceptance mechanism and the Boston mechanism both eliminated significantly more justified envy but they also assigned significantly fewer subjects to their most preferred option.

Figure 9 illustrates the percentage of subjects who accurately reported their preferences over time during a preference reporting period. The first graph illustrates the empirical results under the student optimal deferred acceptance mechanisms. The second graph illustrates the empirical results under the Boston mechanism. The third graph illustrates the empirical results under the top trading cycles mechanism. In all three graphs, the blue line illustrates the empirical path of the truthful reporting percentage under continuous feedback throughout the reporting period. The green line illustrates the empirical path of the truthful reporting percentage under conventional discrete feedback at the end of each reporting period. The vertical axis of each graph indicates the percentage of participants who accurately reported their preferences. The horizontal axis of each graph indicates seconds within the preference reporting period.

Under the student optimal deferred acceptance mechanism subjects selected truthful preference reports about equally often under both continuous and discrete feedback. Under the top trading cycles mechanism mechanism subjects were slightly less likely to accurately report their preferences under continuous assignment feedback. These results are consistent with theoreti-

[18] See Figure 9 for more details regarding the empirical truthful reporting dynamics.

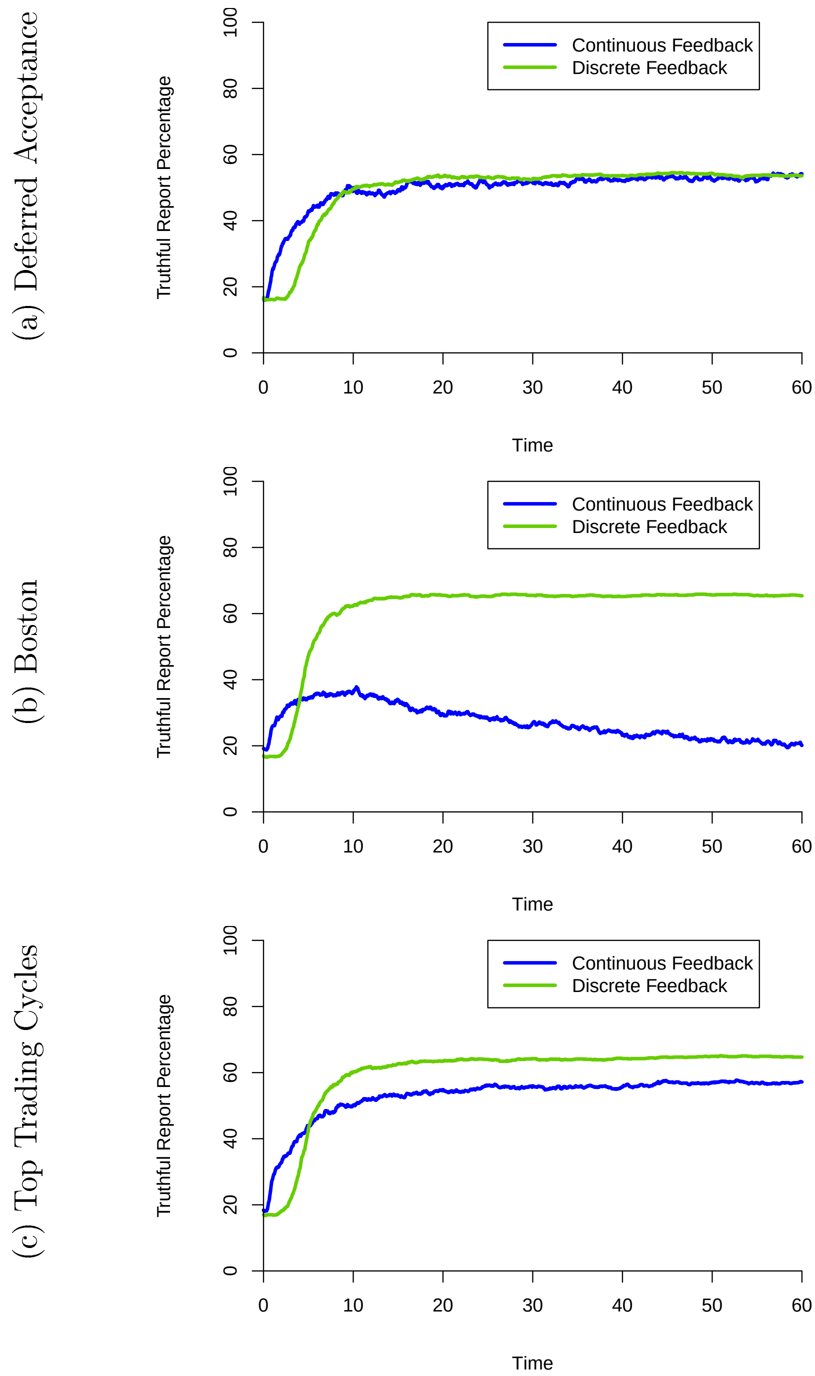

Figure 9: Percentage of truthful preference reports over time within a period. The first graph illustrated the empirical results under the student optimal deferred acceptance mechanisms. The second graph illustrates the empirical results under the Boston mechanism. The third graph illustrates the empirical results under the top trading cycles mechanism.

cal predictions from the adaptive best response[19] dynamic. As illustrated by Figure 1, adaptive adjustment to myopic best responses produces convergence to equilibrium assignments under both of the strategy proof mechanisms, but as illustrated by Figure 2, this adaptive adjustment process does not converge to accurate preference reporting behavior. In line with these theoretical predictions, subjects in these strategy proof mechanisms were significantly more likely to achieve equilibrium assignments when they received feedback during the preference reporting period, but they were not significantly more likely to report their preferences accurately.

In contrast, under the manipulable Boston mechanisms, subjects were significantly more likely to report their preferences truthfully under conventional discrete assignment feedback than under continuous assignment feedback, consistent with theoretical predictions from the adaptive best response dynamic.. Unlike the strategy proof deferred acceptance and top trading cycles mechanisms, the Boston mechanisms is manipulable, so it has no dominant strategy Nash equilibrium under which participants accurately report their preferences. Further, in the Boston mechanism, participants generally misreport their preferences under the Nash equilibrium. Hence, the lower percentage of accurate preference reports in the Boston mechanism under the continuous feedback is consistent with the presence of more equilibrium behavior under continuous feedback. Under discrete feedback, about two-thirds[20] of the subject populations naively selected truthful preference reports and the equilibrium assignment percentage decreased[21] over the reporting period, suggesting that discrete-feedback was insufficient for subjects to develop strategic preference reports. In contrast, under continuous assignment feedback, only about one-fifth of the subjects in the Boston mechanism selected truthful preference

[19] See Subsection 2.3 for more details on the adaptive best response dynamic.
[20] See Figure 9 for more details regarding the empirical dynamics of truthful reporting. [21] See Figure 6 for more details regarding the equilibrium assignment dynamics.

reports and the the equilibrium assignment percentage increased over the preference reporting period, suggesting that continuous-feedback helped subjects to develop strategic preference reports.

Under all three school choice mechanisms, subjects achieved more equilibrium assignments when they had access to continuous assignment feedback. In equilibrium, the Deferred acceptance mechanism and the Boston mechanism fully eliminate justified envy but they do not assign any participants in the experimentally investigated school choice environment to their most preferred school. Accordingly, both of these mechanisms eliminated significantly more justified envy but assigned significantly fewer subjects to their most preferred option when subjects received assignment feedback throughout the preference reporting period. In contrast, the dominant strategy Nash equilibrium of the top trading cycles mechanism is Pareto optimal but it fails to eliminate justified envy. Accordingly, it assigned significantly more subjects to their most preferred option when subjects received assignment feedback throughout the preference reporting period.

6 Conclusion

Classical mechanism design theory suggests that strategy proof mechanisms will induce truthful preference reports and achieve equilibrium outcomes. These theoretical predictions are difficult to verify in the field where preferences are unobservable and school choice mechanisms rarely satisfy the exact assumptions of theory. Experimental investigations can help test predictions from mechanism design theory in a controlled environment and provide valuable information regarding the empirical properties of school choice mechanisms.

Previous studies find that even strategy-proof student assignment mechanisms frequently fail achieve equilibrium outcomes. These departures from the dominant strategy Nash equilibrium predictions may result from the presence of boundedly rational participants. Theoretical predictions from the adaptive best response dynamic suggest that the prevision of assignment feedback throughout the preference reporting period may help boundedly rational participants to more reliably achieve equilibrium outcomes in three widely employed school choice mechanisms: the deferred acceptance mechanism, the top trading cycles mechanism, and the Boston mechanism. To test this hypothesis, the experiment investigates novel implementations of these mechanisms that provide participants with assignment feedback throughout the reporting period.

In all three of these school choice mechanisms, subjects achieved equilibrium assignments significantly more often under implementations that provided assignment feedback throughout the preference reporting period than under conventional discrete feedback implementations. Subjects in the top trading cycles mechanism were significantly more likely to receive their most preferred school under continuous assignment feedback. Subjects in the deferred acceptance mechanism and the Boston mechanism were significantly less likely to exhibit justified envy under continuous assignment feedback. Consistent with the theoretical predictions from the adaptive best response dynamics, these experimental results suggest that the provision of assignment feedback throughout the preference reporting period can help promote rational preference revelation behavior by giving participants more opportunity for learning and adjustment.

Student assignment mechanisms impact the well being of children in many school districts throughout the world. For example, the Boston mechanism was originally used in Boston's school choice system. In 2012, the New Orleans recovery school district used an algorithm based on the top trading cycles assignment mechanism (Vanacore, 2012). In 2008, a variation of the student optimal deferred acceptance mechanism was employed in New York City (Roth, 2008). The results of this study point towards new ways of providing feedback in these school choice mechanisms that can more effectively achieve policy goals. However, further investigation is necessary to determine the extent to which these findings extend to field settings.

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